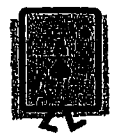
BRUSHING UP on ESSENTIAL ALGEBRA SKILLS

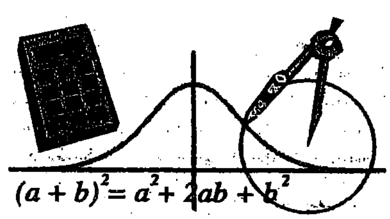
To Get you Ready to



PRE- CALCULUS

Name: ______________________________

Directions: *Use* pencil and the *space* provided next to the question to show all work. The purpose of this packet *is* to give you a review of basic skills. Please refrain from using a calculator!



BRUSHING UP ON BASIC ALGEBRA SKILLS

Name: ______________________________ DUE DATE: __________

Directions: *Use pencil, show work, box in your answers.*

Monomial Factors of Polynomials

A **monomial** is an expression that is either a numeral, a variable, or the product of a numeral and one or more variables. Example of monomials: $7,\ x,\ 6x^2y^3$.

A sum of monomials is called a **polynomial**. Some polynomials have special names:

Binomials (two terms): $3x-5$ *or* $2xy+x^2$

Trinomials (3 terms): $x^2+5x-15$ *or* $x^2-6xy+9y^2$

Divide:

1) $\dfrac{24x-12}{6}$

2) $\dfrac{8x^4-4x^3-6x^2}{-2x^2}$

Factor (monomials only): Example: $15a-25b+35=5(3a-5b+7)$

3) $7a^3-21a^2-14a$

4) $5ax^2-10a^2x+15a^3$

Simplify: Example:

$$\frac{14x-21}{7}-\frac{10x-25}{5}=\frac{7(2x-3)}{7}-\frac{5(2x-5)}{5}=2x-3-2x+5=2$$

Factor

Cancel out common factors
Simplify and combine

5) $\dfrac{6a+9b}{3}-\dfrac{7a+21b}{7}=$

6) $\dfrac{x^2y-3x^2y^2}{xy}+\dfrac{6xy+9xy^2}{3y}=$

Multiplying Binomials Mentally

Write each product as a trinomial:

7) $(x-9)(x+4)=$

8) $(4-x)(1-x)=$

9) $(2a+5)(a-2)=$

10) $(2x-5)(3x+4)=$

Find the values of p, q, and r that make the equation true.

11) $(px+q)(2x+5)=6x^2+11x+r$

Difference of Two Squares

You must use the shortcut below (do not "FOIL"!!!!)

$$(a+b)(a-b)=a^2-b^2$$

$$(First+Second)\times(First-Second)=(First)^2-(Second)^2$$

Write each product as a binomial:

12) $(x+7)(x-7)=$

13) $(y+8)(y-8)=$

14) $(5x+2)(5x-2)=$

15) $(8x-11)(8x+11)=$

16) $(4a+5b)(4a-5b)=$

17) $(x^2-9y)(x^2+9y)=$

Now let's try reversing the process above... Factor:

18) $x^2-36=$

19) $m^2-81=$

20) $25a^2-1=$

21) $49x^2-9y^2=$

Factor each expression as the difference of two squares. Then simplify.

Example: $x^2-(x-3)^2=[x-(x-3)][x+(x-3)]=3(2x-3)$

Apply the formula **simplify**

22) $(x+4)^2-x^2=$

23) $9(x+1)^2-4(x-1)^2=$

Squares of Binomials and Perfect Square Trinomials

Every time you square a binomial, the same pattern comes up. To speed up the process, we should memorize:

$$(a+b)^2 = a^2 + 2ab + b^2 \quad \text{and} \quad (a-b)^2 = a^2 - 2ab + b^2$$

A trinomials is called a perfect square trinomial if it is the square of a binomial. For example, $x^2 - 6x + 9$ is a perfect square trinomial because it is equal to $(x-3)^2$.

Write each square as a trinomial.

24) $(a-9)^2 =$

25) $(x+7)^2 =$

26) $(4x-1)^2 =$

27) $(5a-2b)^2 =$

Decide if each trinomial is a perfect square. If it is, factor it. Otherwise, write **not a perfect square.**

28) $a^2 + 6a + 9 =$

29) $y^2 - 14y + 49 =$

30) $121 - 22x + x^2 =$

31) $9a^2 + 30ab + 100b^2 =$

32) $49x^2 - 28xy + 4y^2 =$

33) $25x^2 - 15xy + 36y^2 =$

34) $a^2b^2 - 12ab + 36 =$

35) $121 - 33x^2 + 9x^4$

36) Show that $a^4 - 8a^2 + 16$ can be factored as $(a+2)^2(a-2)^2$.

37) Solve and check: $(x+2)^2 - (x-3)^2 = 35$

Factoring Quadratic Trinomials

To factor a trinomial of the form $x^2 + bx + c$, you must find two numbers, r and s, whose product is c and whose sum is b.

$$x^2 + bx + c = (x + r)(x + s)$$

When you find the product $(x + r)(x + s)$ you obtain

$$x^2 + bx + c = x^2 + (r + s)x + rs$$

Example: Factor $x^2 - 2x - 15$.

a. List the factors of -15 (the last term).

b. Either write them down or do this mentally. Find the pairs of factors with sum -2 (the middle term).

Factors of -15	Sum of the factors
1, -15	-15 *(discard)*
-3, 5	2 (discard)
3, -5	-2 (keep)

$\therefore x^2 - 2x - 15 = (x - 5)(x + 3)$

Check the result by multiplying...

Factor. Check by multiplying (mentally).

38. $x^2 + 8x + 12 =$ ______________
39. $x^2 - 7x + 12 =$ ______________
40. $x^2 - 4x + 12 =$ ______________
41. $x^2 - 9x + 18 =$ ______________
42. $x^2 - 3x - 18 =$ ______________
43. $x^2 + 11x + 18 =$ ______________
44. $x^2 - 5x - 36 =$ ______________
45. $x^2 - 15x + 36 =$ ______________
46. $x^2 - 9x - 36 =$ ______________
47. $x^2 + 3x - 28 =$ ______________

Factoring General Quadratic Trinomials of the type ax^2+bx+c

Example: Factor $2x^2+7x-9 \;=\; (2x+9)(x-1)$

List all factors Of $2x^2$ — List factors of -9

Factor:

48) $3x^2+7x+2$

49) $2x^2+5x+3$

50) $2x^2-15x+7$

51) $3a^2+4a-4$

52) $5a^2-6a-2$

53) $3x^2-2x-5$

54) $3m^2+7m-6$

55) $4a^2-a-3$

Factor by Grouping

Example 1: $7(a-2)+3a(a-2)=(7+3a)(a-2)$

Notice that we "factored out" the common factor (a - 2)

Example 2: $5(x-3)-2x(3-x)$

Notice that $x-3$ *and* $3-x$ *are opposites.*

$$5(x-3)-2x(3-x)=5(x-3)+2x(x-3)=(5+2x)(x-3)$$

56) $2x(x-y)+y(y-x)$

57) $3a(2b-a)-2b(a-2b)$

Group and Factor:

58) $3a+ab+3c+bc$

59) $3a^3+a^2+6a+2$

60) $3ab-b-4+12a$

Solving Equations by Factoring – The Zero Product Property

Key Concept:

Zero-Product Property

For all real numbers a and b:

$a \cdot b = 0$ if and only if $a = 0$ or $b = 0$

A product of factors is equal to zero if and only if at least one of the factors is 0.

Solve:

61) $(x+5)(x-3)=0$

62) $2x(x-9)=0$

63) $(2a-3)(3a+2)=0$

64) $3x(5x+2)(x-7)=0$

65) $a^2-3a+2=0$

66) $x^2-12x+35=0$

67) $b^2=4b+32$

68) $25x^2-16=0$

69) $7x^2=18x-11$

70) $8y^3-2y^2=0$

71) $4x^3-12x^2+8x=0$

72) $9x^3+25x=30x^2$

Sample for items 73-74: $(a-1)(a+3)=12$

$a^2+2a-3-12=0$ expand the left side; bring over 12; set it = 0;

$(a-3)(a+5)=0$ combine like terms; factor it;

$a=3$ *or* $a=-5$

73) $(x+1)(x-5)=16$

74) $(2z-5)(z-1)=2$

Simplifying Fractions - Follow the 2 examples below.

Example 1: Simplify $\frac{3x+6}{3x+3y}$.

Solution: $\frac{3x+6}{3x+3y}=\frac{3(x+2)}{3(x+y)}$ Factor the numerator and denominator; look for common factors;

$=\frac{x+2}{x+y},\ x\neq -y$ Cancel out common factor which is 3;

Example 2: Simplify $\frac{x^2-9}{(2x+1)(3-x)}$

Solution: $\frac{x^2-9}{(2x+1)(3-x)}=\frac{(x+3)(x-3)}{-(2x+1)(x-3)}$ First factor the numerator; "pull out" a negative in the factor (3-x) to make it a common factor with the numerator;

$=-\frac{x+3}{2x+1},\left(x\neq -\frac{1}{2},\ x\neq 3\right)$ exclude the first as it would make the denominator =0; exclude the 2nd as it would make both numerator and denominator =0.

Simplify. Give any restrictions on the variable.

75) $\frac{5x-10}{x-2}$

76) $\frac{2a-4}{a^2-4}$

77) $\frac{(x-5)(2+7x)}{(5-x)(7x+2)}$

78) $\frac{x^2+8x+16}{16-x^2}$

Multiplying Fractions

Multiplication Rule for Fractions

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$ To multiply fractions, you multiply their numerators and multiply their denominators.

Note: You can multiply first and then simplify, or you can simplify first and then multiply.

Example: $\frac{x^2 - x - 12}{x^2 - 5x} \cdot \frac{x^2 - 25}{x + 3} = \frac{(x-4)(x+3)}{x(x-5)} \cdot \frac{(x-5)(x+5)}{x+3}$

$= \frac{(x-4)(x+5)}{x}$ Notice how common factors were cancelled.

Simplify.

79) $\frac{a+2}{a^2} \cdot \frac{3a}{a^2 - 4}$

80) $\frac{a^2 - x^2}{a^2} \cdot \frac{a}{3x - 3a}$

81) A triangle has base $\frac{3x}{4}$ cm and height $\frac{8}{9x}$ cm. What is its area?

Dividing Fractions

Division Rule for Fractions: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$ *Example*: $\frac{7}{9} \div \frac{2}{3} = \frac{7}{9} \cdot \frac{3}{2} = \frac{7}{6}$.

To divide by a fraction, you multiply by its reciprocal!

82) $\frac{2+2a}{6} \div \frac{1+a}{9} =$

83) $\frac{x^2-1}{2} \div \frac{x+1}{16} =$

84) $\frac{2a+2b}{a^2} \div \frac{a^2-b^2}{4a}$

85) $\frac{4x^2-25}{x^2-16} \div \frac{12x+30}{2x^2+8x} =$

Adding and Subtracting Algebraic Fractions

Key Steps: (1) Find the Least Common Denominator (LCD);
(2) Re-write each fraction being added or subtracted with the same common denominator.
(3) Add or subtract their numerators and write the result over the common denominator.

Example:

$$\frac{3}{6x-30} + \frac{8}{9x-45} = \frac{3}{6(x-5)} + \frac{8}{9(x-5)}$$ Factor out denominators to more easily identify LCD

$$= \frac{9}{18(x-5)} + \frac{16}{18(x-5)}$$ multiply the first fraction by 3, the second by 2

$$= \frac{25}{18(x-5)} \text{ or } \frac{25}{18x-90}$$

86) $\frac{4x+3}{3} - \frac{7x}{4} + \frac{x-3}{6} =$

87) $\frac{2}{x-3}+\frac{4}{x+3}=$

88) $\frac{x}{x^2-1}+\frac{4}{x+1}=$

89) $\frac{3a}{a-2b}+\frac{6b}{2b-a}=$

90) $\frac{x-11}{x^2-9}-\frac{x-7}{x^2-3x}=$